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# Numerical solution of hyperbolic heat conduction in thin surface layers

Tzer-Ming Chen *

*Department of Vehicle Engineering, National Taipei University of Technology, No. 1, Sec. 3, Chung-Hsiao East Road, Taipei 10643, Taiwan, ROC*

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**Abstract**

The purpose of the present paper is to propose a new hybrid method investigating the effect of the surface curvature of a solid body on hyperbolic heat conduction. The difficulty encountered in the numerical solutions of hyperbolic heat conduction problems is the numerical oscillation in vicinity of sharp discontinuities. In the present study, we have developed a new hybrid method combined the Laplace transform, the weighting function scheme [Shong-leih Lee, Weighting function scheme and its application on multidimensional conservation equations, Int. J. Heat Mass Transfer 32 (1989) 2065–2073], and the hyperbolic shape function for solving time dependent hyperbolic heat conduction equation with a conservation term. Four different examples have been analyzed by the present method. It is found from these examples that the present method is in good agreement in the analytical solutions [Tsai-tse Kao, Non-Fourier heat conduction in thin surface layers, J. Heat Transfer 99 (May) (1977) 343–345] and does not exhibit numerical oscillations at the wave front and the surface temperature is modified by the surface curvature during the short period when the non-Fourier effect is significant. The curvature will increase or decrease the temperature of the wave front, depending on whether the surface is concave or convex.
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*Keywords:* Hyperbolic heat conduction; Hybrid method; Thin surface layers

## 1. Introduction

In the last decades, study of the hyperbolic heat conduction equation has received considerable interest, because of it wide applicability in engineering applications, such as laser-aided material processing, cryogenic engineering, the high-intensity electromagnetic irradiation of a solid and the high-rate heat transfer in rarefied media. The solutions of the hyperbolic heat conduction can be found in a number of publications, such as one-dimensional given by [2–13] and two-dimensional solutions given by [14–16]. Baumeiser and Hamill [2], Taitel [3], Ozisik and Vick [4], and Wu [5] obtained an analytical solution of one-dimensional HHC, for a semi-infinite medium or in a finite medium with convection, or radiation at the wall surface. Carey and Tai [6] applied the central and backward difference schemes to examine the oscillation of numerical solution at the reflected boundary. To remedy the numerical difficulty encountered, many numerical schemes such as the predictor-corrector scheme [7], the transfinite element formulation [8], the technique based on the Galerkin finite element and mixed implicit-explicit scheme [9], the characteristic method [10], and the hybrid scheme [11] have been proposed. Glass et al. [12] and Yeung and Tung [13] studied the effect of the surface radiation on thermal wave propagation in a one-dimensional slab.

The objective of the present study is to propose a new hybrid method investigating the influence of the surface curvature of a solid body on hyperbolic heat conduction. The present method combine the Laplace transform, weighting function scheme, and the hyperbolic shape function for solving time dependent hyperbolic heat conduction equation with a conservation term. The Laplace transfer method is used to remove the time-dependent terms from the governing equation, and then the discretized expression of the transformed equation is given by the weighting function scheme and the hyperbolic shape function. It is found from four examples that the present method is in good

* Tel.: +886 02 29912191.
*E-mail address:* tmchen@ntut.edu.tw

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**Nomenclature**

| | |
|---|---|
| $C$ | propagation velocity of thermal wave |
| $c_p$ | specific heat |
| $E_r$ | dimensionless radiation parameter $\frac{\alpha_s \sigma \alpha^4 f_r^3}{k^4 c^4}$ |
| $f_r$ | reference heat flux |
| $k$ | thermal conductivity |
| $q$ | heat flux |
| $R$ | the average radius of curvature at $x=0$, $y=0$ |
| $R_1, R_2$ | the principal radii of curvature at $x=0$, $y=0$ |
| $s$ | Laplace transform parameter |
| $T$ | temperature |
| $T_0$ | surrounding temperature |
| $W_f(z)$ | weighting function, $\frac{z}{1-e^{-z}}$ |
| $z$ | parameter of the weighting function |
| $x, y, z$ | coordinators |

*Greek symbols*

| | |
|---|---|
| $\alpha$ | thermal diffusivity, $\frac{k}{\rho c_p}$ |
| $\alpha_s$ | surface absorptivity |
| $\gamma$ | $\gamma = \left(\frac{\partial^2 \varsigma}{\partial x^2} + \frac{\partial^2 \varsigma}{\partial y^2}\right)_{x=0,y=0} = -\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$ |
| $\eta$ | dimensionless length, $\frac{c\varsigma}{2\alpha}$ |
| $\theta$ | dimensionless temperature, $\frac{(T-T_0)kc}{\alpha f_r}$ |
| $\widehat{\theta}$ | the previously calculated surface temperature |
| $\rho$ | density |
| $\sigma$ | Stefan–Boltzmann constant |
| $\varsigma$ | length, $z - f(x,y)$ |
| $\xi$ | dimensionless time, $\frac{c^2 t}{2\alpha}$ |

*Superscript*

| | |
|---|---|
| – | the Laplace transform |

agreement in the analytical solutions [1] and does not exhibit numerical oscillations at the wave front and the surface temperature is modified by the surface curvature during the short period when the non-Fourier effect is significant.

## 2. Analysis

Consider a coordinate system, as shown as in Fig. 1. The $x$–$y$ plane forms a tangential plane at the surface point of interest. The surface of the body can be described by an equation of the form $z = f(x,y)$. The hyperbolic heat conduction equation is given by

$$\frac{1}{C^2}\frac{\partial^2 T}{\partial t^2} + \frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \tag{1}$$

Let us introduce a new independent variable $\varsigma = z - f(x,y)$ and neglect terms of order $(\frac{\delta}{R})^2$, where $\delta$ is the heat penetration length and $R$ is the average radius of curvature at $x=0$, $y=0$. The equation at $x=0$ and $y=0$ is given by

$$\frac{1}{C^2}\frac{\partial^2 T}{\partial t^2} + \frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial \varsigma^2} + \gamma\frac{\partial T}{\varsigma} \tag{2}$$

where

$$\gamma = \left(\frac{\partial^2 \varsigma}{\partial x^2} + \frac{\partial^2 \varsigma}{\partial y^2}\right)_{x=0,y=0} = -\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \tag{3}$$

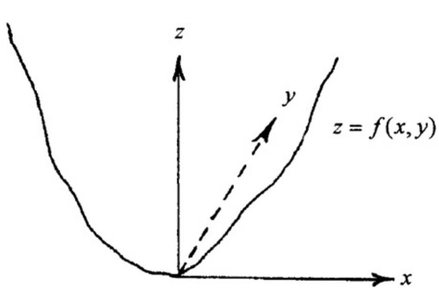


Fig. 1. Coordinate system.

$R_1$ and $R_2$ are the principal radii of curvature at $x=0$, $y=0$.

For convenience of numerical analysis, let us define by the following dimensionless variables:

$$\xi = \frac{C^2 t}{2\alpha} \tag{4}$$

$$\eta = \frac{C\varsigma}{2\alpha} \tag{5}$$

$$\theta(\eta, \xi) = \frac{kC(T - T_0)}{\alpha f_r} \tag{6}$$

$$Q(\eta, \xi) = \frac{q}{f_r} \tag{7}$$

The resulting equation becomes

$$\frac{\partial^2 \theta}{\partial \xi^2} + 2\frac{\partial \theta}{\partial \xi} = \frac{\partial^2 \theta}{\partial^2 \eta} + \varepsilon\frac{\partial \theta}{\partial \eta} \tag{8}$$

where

$$\varepsilon = \frac{2\gamma\alpha}{C} \tag{9}$$

## 3. Numerical scheme

To remove the $\xi$-dependent terms, taking the Laplace transform of Eq. (8) with respect to $\xi$ gives

$$\frac{d^2\bar{\theta}}{d\eta^2} + \varepsilon\frac{d\bar{\theta}}{d\eta} - (s^2 + 2s)\bar{\theta} = 0 \tag{10}$$

Consider a homogeneous second-order ordinary differential of the form

$$\theta'' + a\theta' - \lambda^2\theta = 0 \tag{11}$$

Combining the weighting function scheme and hyperbolic shape function, the resulting discretized form is given

$$(\theta'' + a\theta' - \lambda^2\theta)_i = a_W\theta_{i-1} + a_P\theta_i + a_E\theta_{i+1} \tag{12}$$

where

$$a_W = \frac{\lambda W(-z_{i-1})}{\Delta x_{i-1}\sinh(\lambda\Delta x_{i-1})} \tag{13}$$

$$a_E = \frac{\lambda W(z_i)}{\Delta x_i \sinh(\lambda\Delta x_i)} \tag{14}$$

$$a_P = -a_W - a_E + \frac{2\lambda(\cosh(\lambda\Delta x_i) - 1)}{\Delta x_i \sinh(\lambda\Delta x_i)} \tag{15}$$

and

$$W(z) = \frac{z}{1 - e^{-z}} \tag{16}$$

$$z_i = a_{i+\frac{1}{2}}\Delta x_i \tag{17}$$

Using these Eqs. (12)–(17) for Eq. (10) leads to the discretized expression of Eq. (10) as

$$a_W\theta_{i-1} + a_P\theta_i + a_E\theta_{i+1} = 0 \tag{18}$$

$$a_W = \frac{(s^2+2s)^{\frac{1}{2}}W(-z_{i-1})}{\Delta x_{i-1}\sinh\left[(s^2+2s)^{\frac{1}{2}}\Delta x_{i-1}\right]} \tag{19}$$

$$a_E = \frac{(s^2+2s)^{\frac{1}{2}}W(z_i)}{\Delta x_i\sinh\left[(s^2+2s)^{\frac{1}{2}}\Delta x_i\right]} \tag{20}$$

$$a_P = -a_W - a_E + \frac{2(s^2+2s)^{\frac{1}{2}}\{\cosh[(s^2+2s)^{\frac{1}{2}}\Delta x_i] - 1\}}{\Delta x_i\sinh\left[(s^2+2s)^{\frac{1}{2}}\Delta x_i\right]} \tag{21}$$

$$z_i = \varepsilon\Delta x_i \tag{22}$$

The rearrangement of Eqs. (18)–(22) in conjunction with the boundary conditions can yield the following matrix system

$$[K]\{\bar{\theta}\} = \{f\} \tag{23}$$

The nodal dimensionless temperature $\theta_i$ can be determined by using the application of the Gaussian elimination algorithm and the numerical inversion of the Laplace transform technique [17].

## 4. Results and discussion

**Example 1.** Prescribed wall temperature. The initial and boundary conditions for this case are given by

$$\theta(\eta, 0) = 0, \quad \frac{\partial\theta}{\partial\xi}(\eta, 0) = 0 \tag{24}$$

$$\theta(0, \xi) = 1, \quad \theta(\eta \to \infty, \xi) = 0 \tag{25}$$

The analytical solution [1] of this example is expressed as

$$\theta(\eta,\xi) = e^{\frac{-\eta}{2\varepsilon}}\left\{e^{-\eta} + \left(1-\frac{\varepsilon^2}{4}\right)^{\frac{1}{2}}\eta\int_\eta^\xi e^{-\tau}\frac{I_1\left\{\left[(1-\frac{\varepsilon^2}{4})(\tau^2-\eta^2)\right]^{\frac{1}{2}}\right\}}{(\tau^2-\eta^2)^{\frac{1}{2}}}d\tau\right\}U(\xi-\eta) \tag{26}$$

Table 1 lists the comparison of the present method solutions by $\Delta\eta = 0.025$ and analytical solutions for the problem at $t = 0.5$ and $t = 1.0$. From Table 1, it is seen that

Table 1
Comparison of the present method and analytical solution resulting from a prescribed wall temperature

| $x$ | Present method | | | Analytic solution Eq. (26) | | |
|---|---|---|---|---|---|---|
| | $\varepsilon = -0.1$ | $\varepsilon = 0.0$ | $\varepsilon = 0.1$ | $\varepsilon = -0.1$ | $\varepsilon = 0.0$ | $\varepsilon = 0.1$ |
| *t = 0.5* | | | | | | |
| 0.0 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 |
| 0.1 | 0.924486 | 0.919913 | 0.915287 | 0.924486 | 0.919913 | 0.915287 |
| 0.2 | 0.848567 | 0.840177 | 0.831764 | 0.848566 | 0.840177 | 0.831763 |
| 0.3 | 0.772592 | 0.761140 | 0.749758 | 0.772591 | 0.761140 | 0.749758 |
| 0.4 | 0.696914 | 0.683146 | 0.669587 | 0.696914 | 0.683146 | 0.669587 |
| 0.5 | 0.310868 | 0.303265 | 0.295707 | 0.310943 | 0.303265 | 0.295778 |
| 0.6 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 0.7 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 0.8 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 0.9 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 1.0 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| *t = 1.0* | | | | | | |
| 0.0 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 |
| 0.1 | 0.937279 | 0.932676 | 0.927953 | 0.937279 | 0.932676 | 0.927952 |
| 0.2 | 0.874187 | 0.865609 | 0.856877 | 0.874186 | 0.865609 | 0.856876 |
| 0.3 | 0.810980 | 0.799056 | 0.787011 | 0.810978 | 0.799056 | 0.787010 |
| 0.4 | 0.747915 | 0.733268 | 0.718589 | 0.747914 | 0.733268 | 0.718587 |
| 0.5 | 0.685251 | 0.668492 | 0.651831 | 0.685250 | 0.668492 | 0.651830 |
| 0.6 | 0.623244 | 0.604968 | 0.586949 | 0.623243 | 0.604968 | 0.586948 |
| 0.7 | 0.562146 | 0.542929 | 0.524142 | 0.562145 | 0.542929 | 0.524141 |
| 0.8 | 0.502205 | 0.482597 | 0.463593 | 0.502204 | 0.482597 | 0.463592 |
| 0.9 | 0.443659 | 0.424181 | 0.405474 | 0.443660 | 0.424179 | 0.405469 |
| 1.0 | 0.193172 | 0.183940 | 0.174789 | 0.193370 | 0.183940 | 0.174969 |
| 1.1 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 1.2 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |

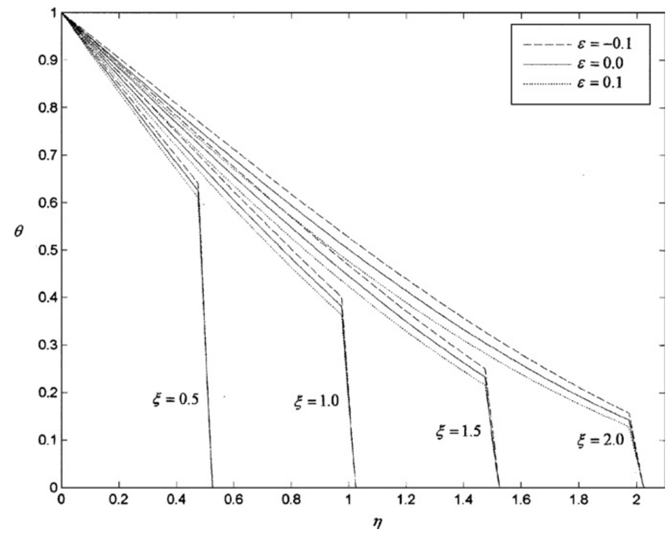


Fig. 2. The influence of the surface curvature of a solid body on hyperbolic heat conduction problem with a prescribed wall temperature by $\Delta\eta = 0.025$ at $t = 0.5$, $t = 1.0$, $t = 1.5$ and $t = 2.0$.

the present method solutions are in agreement with the analytical solution using the Eq. (26). Fig. 2 shows the influence of the surface curvature of a solid body on hyperbolic heat conduction problem with a prescribed wall temperature by $\Delta\eta = 0.025$ at $t = 0.5$, $t = 1.0$, $t = 1.5$, and $t = 2.0$. The curvature will increase or decrease the temperature of the wave front, depending on whether the surface is concave or convex.

**Example 2.** Prescribed wall heat flux. The initial and boundary conditions for this case are given by

$$\theta(\eta, 0) = 0, \quad \frac{\partial\theta(\eta, 0)}{\partial\xi} = 0 \tag{27}$$

$$Q(0, \xi) = 1, \quad Q(\eta \to \infty, \xi) = 0, \quad Q(\eta, 0) = 0 \tag{28}$$

The analytical solution [1] of this example is expressed as

$$\begin{aligned}\theta(\eta, \xi) = {} & \frac{\alpha}{C} e^{-\frac{\varepsilon}{2\eta}} \left\{ e^{-\xi} I_0 \left\{ \left[ \left(1 - \frac{\varepsilon^2}{4}\right)(\xi^2 - \tau^2) \right]^{\frac{1}{2}} \right\} \right. \\ & \left. + \int_\eta^\xi e^{-\tau} I_0 \left\{ \left[ \left(1 - \frac{\varepsilon^2}{4}\right)(\tau^2 - \eta^2) \right]^{\frac{1}{2}} \right\} d\tau \right\} U(\xi - \eta) \\ & - \frac{\alpha}{C} \varepsilon \left\{ e^{-\eta} \int_\eta^\infty e^{-\frac{\varepsilon}{2\zeta}} I_0 \left\{ \left[ \left(1 - \frac{\varepsilon^2}{4}\right)(\xi^2 - \zeta^2) \right]^{\frac{1}{2}} \right\} U(\xi - \zeta)\, d\zeta \right. \\ & \left. + \int_\eta^\infty e^{-\frac{\varepsilon}{2\zeta}} \int_0^\xi e^{-\zeta} I_0 \left\{ \left[ \left(1 - \frac{\varepsilon^2}{4}\right)(\tau^2 - \zeta^2) \right]^{\frac{1}{2}} \right\} U(\tau - \zeta)\, d\tau\, d\zeta \right\}\end{aligned} \tag{29}$$

The boundary condition for the Laplace transform of the dimensionless temperature at surface $\eta = 0$ can be obtained

$$\frac{d\bar{\theta}}{d\eta}(0, s) = -\frac{s + 2}{s} \tag{30}$$

Table 2
Comparison of the present method solutions and analytical solutions resulting from a prescribed wall heat flux

| x | Present method | | | Analytic solution Eq. (29) | | |
|---|---|---|---|---|---|---|
| | ε = −0.1 | ε = 0.0 | ε = 0.1 | ε = −0.1 | ε = 0.0 | ε = 0.1 |
| *t = 0.5* | | | | | | |
| 0.0 | 1.471635 | 1.446491 | 1.421627 | 1.471626 | 1.446491 | 1.421626 |
| 0.1 | 1.277607 | 1.252940 | 1.228559 | 1.277603 | 1.252940 | 1.228560 |
| 0.2 | 1.095597 | 1.072261 | 1.049251 | 1.095592 | 1.072261 | 1.049248 |
| 0.3 | 0.925646 | 0.904386 | 0.883482 | 0.925641 | 0.904386 | 0.883482 |
| 0.5 | 0.311089 | 0.303265 | 0.295179 | 0.310942 | 0.303265 | 0.295778 |
| 0.6 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 0.7 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 0.8 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 0.9 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 1.0 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| *t = 1.0* | | | | | | |
| 0.0 | 1.863602 | 1.813100 | 1.763559 | 1.863574 | 1.813100 | 1.763574 |
| 0.1 | 1.668825 | 1.618795 | 1.569746 | 1.668799 | 1.618795 | 1.569761 |
| 0.2 | 1.484548 | 1.435864 | 1.388206 | 1.484523 | 1.435864 | 1.388219 |
| 0.3 | 1.310795 | 1.264251 | 1.218782 | 1.310799 | 1.264251 | 1.218775 |
| 0.4 | 1.147647 | 1.103865 | 1.061213 | 1.147639 | 1.103865 | 1.061217 |
| 0.5 | 0.995030 | 0.954580 | 0.915304 | 0.995021 | 0.954580 | 0.915306 |
| 0.6 | 0.852891 | 0.816236 | 0.780764 | 0.852886 | 0.816236 | 0.780768 |
| 0.7 | 0.721143 | 0.688636 | 0.657303 | 0.721138 | 0.688636 | 0.657304 |
| 0.8 | 0.599651 | 0.571554 | 0.544583 | 0.599648 | 0.571554 | 0.544584 |
| 0.9 | 0.488251 | 0.464732 | 0.442254 | 0.488241 | 0.464732 | 0.442255 |
| 1.0 | 0.193488 | 0.183940 | 0.174648 | 0.193370 | 0.183939 | 0.174969 |
| 1.1 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 1.2 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |

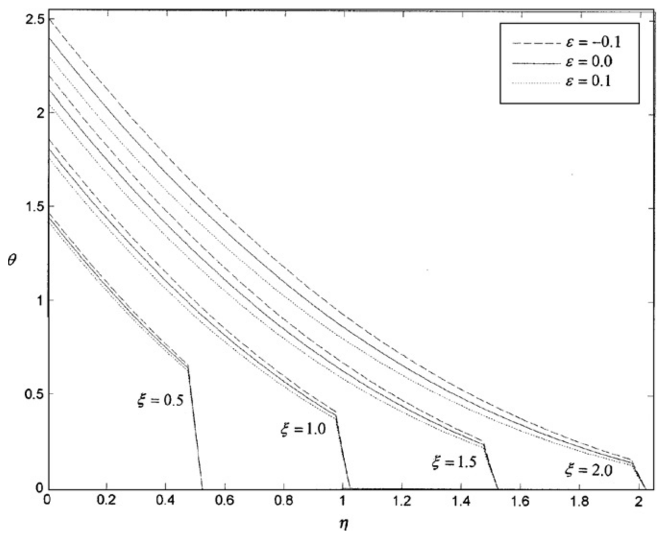


Fig. 3. The influence of the surface curvature of a solid body on hyperbolic heat conduction problem with a prescribed wall heat flux by $\Delta\eta = 0.025$ at $t = 0.5$, $t = 1.0$, $t = 1.5$ and $t = 2.0$.

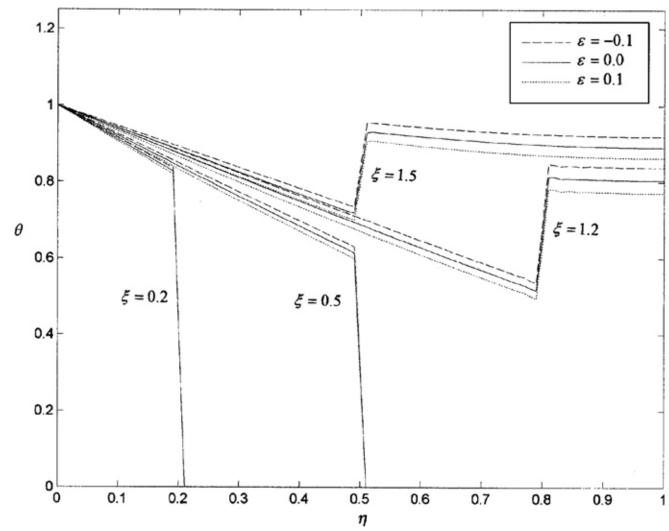


Fig. 4. The effect of the surface curvature of a finite slab body on hyperbolic heat conduction problem by $\Delta\eta = 0.01$ at $t = 0.2$, $t = 0.5$, $t = 1.2$ and $t = 1.5$.

And the discretized form is represented as

$$\frac{(s^2+2s)^{\frac{1}{2}}W(z_1)}{\sinh\left[(s^2+2s)^{\frac{1}{2}}\Delta x_1\right]}\left\{-\cosh[(s^2+2s)^{\frac{1}{2}}\Delta x_1]\bar{\theta}_1+\bar{\theta}_2\right\} = -\frac{s+2}{s} \tag{31}$$

Table 2 shows the comparison of the present method solutions by $\Delta\eta = 0.025$ and analytical solutions [1] resulting from a prescribed wall heat flux problem at $t = 0.5$ and $t = 1.0$. It is observed that the present method solutions well agree with the analytical solutions.

Fig. 3 represents the influence of the surface curvature of a solid body on hyperbolic heat conduction problem with a prescribed wall heat flux by $\Delta\eta = 0.025$ at $t = 0.5$, $t = 1.0$, $t = 1.5$, and $t = 2.0$. It can be seen that the present method solutions do not exhibit numerical oscillations at the wave front.

**Example 3.** Prescribed in a finite slab. The initial and boundary conditions for this case are given by

$$\theta(\eta,0)=0,\quad \frac{\partial\theta}{\partial\xi}(\eta,0)=0 \tag{32}$$

$$\theta(0,\xi)=1,\quad \frac{\partial\theta}{\partial\eta}(1,\xi)=0 \tag{33}$$

Fig. 4 illustrates the effect of the surface curvature of a finite slab body on hyperbolic heat conduction problem by $\Delta\eta = 0.01$ at $t = 0.2$, $t = 0.5$, $t = 1.2$ and $t = 1.5$.

**Example 4.** Prescribed surface radiation. The initial and boundary conditions for this case are given by

$$\theta(\eta,0)=0,\quad \frac{\partial\theta(\eta,0)}{\partial\xi}=0 \tag{34}$$

$$Q(0,\xi)=-E_r\theta^4+1,\quad Q(\eta\to\infty,\xi)=0,\quad Q(\eta,0)=0,$$
$$\text{where } E_r=\frac{\alpha_s\sigma\alpha^4 f_r^3}{k^4c^4} \tag{35}$$

The boundary condition is linearized by the Taylor's series approximation and the Laplace transform of the dimensionless temperature at surface $\eta = 0$ can be obtained

$$\frac{d\bar{\theta}}{d\eta}(0,s)=-(s+2)\left[-E_r\left(4\widehat{\theta}^3\bar{\theta}-\frac{3\widehat{\theta}^4}{s}\right)+\frac{1}{s}\right] \tag{36}$$

where $\widehat{\theta}$ is the previously calculated surface temperature.

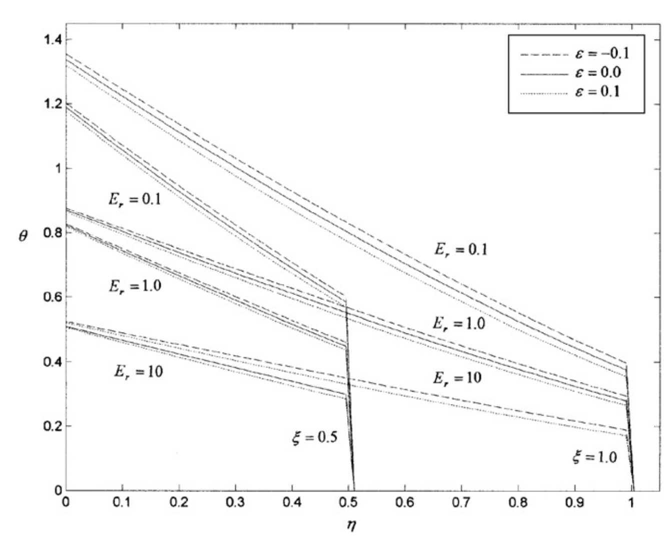


Fig. 5. The influence of the surface curvature of a solid body on hyperbolic heat conduction problem with a prescribed surface radiation by $\Delta\eta = 0.015$, $E_r = 0.1$, $E_r = 1.0$ and $E_r = 10$ at $t = 0.5$ and $t = 1.0$.

And the discretized form is represented as

$$\frac{(s^2+2s)^{\frac{1}{2}}W(z_1)}{\sinh\left[(s^2+2s)^{\frac{1}{2}}\Delta x_1\right]}\left\{-\cosh\left[(s^2+2s)^{\frac{1}{2}}\Delta x_1\right]\bar{\theta}_1+\bar{\theta}_2\right\}$$

$$=(s+2)\left[-E_r\left(4\widehat{\theta}^3\bar{\theta}-\frac{3\widehat{\theta}^4}{s}\right)+\frac{1}{s}\right] \quad (37)$$

Fig. 5 shows the influence of the surface curvature of a solid body on hyperbolic heat conduction problem with a prescribed surface radiation by $\Delta\eta = 0.015$, $E_r = 0.1$, $E_r = 1.0$, and $E_r = 10$ at $t = 0.5$ and $t = 1.0$.

## 5. Conclusions

The new hybrid has shown success in solving the hyperbolic heat conduction problem with a conservation term. To illustrate the accuracy and efficiency of the new method, four different examples have been analyzed. It is found from these examples that the present method is in good agreement in the analytical solutions [1] and does not exhibit numerical oscillations at the wave front. And the influence of surface curvature of a solid surface with hyperbolic heat conduction is also shown in Figs. 2–5. These results represent how the surface temperature is modified by the surface curvature during the short period when the non-Fourier effect is significant. The curvature will increase or decrease the temperature of the wave front, depending on whether the surface is concave or convex.

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